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# RELAXATION OF ROTATIONAL ENERGY IN SUPERSONIC GAS-DYNAMIC JET EXPANSION INTO VACUUM

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**Introduction.** Expansion of a supersonic gas-dynamic jet into vacuum is of interest for studying relaxation of translational and rotational energy under conditions of transition from continual flow to almost free molecular flow. Translational relaxation in such jets has been adequately studied [1–4] (which, in the case of monatomic gases and their mixtures, has made it possible to obtain information on the elastic intermolecular interaction potential in the range of low energies from the experimental dependences of jet parameters on source conditions), whereas the papers devoted to the theoretical investigation of rotational relaxation in a jet are few in number [5–8]. Thus, for example, Lang [7] and Randenija and Smith [8] studied the evolution of parallel, perpendicular, and rotational temperatures in a jet by solving numerically the system of moment equations obtained using the Van Chang–Uhlenbeck kinetic equation. Willis and Hamel [5] examined a similar problem using a model representation of the integral of collisions and asymptotic analysis of momentum equations.

The goal of the present work is to perform a step-by-step asymptotic analysis of the system of momentum equations that follows from the Van Chang–Uhlenbeck kinetic equation for a polyatomic gas in the approximation of a weak nonequilibrium jet. This makes it possible to obtain analytic dependences of the limiting parallel $T_{\|}$, perpendicular $T_{\perp}$, and rotational $T_r$ temperatures on source conditions and on the rotational number of collisions $Z$, which characterizes the inelastic collision of molecules. Comparison between calculated and experimental results allows one to determine values of $Z$ at low temperatures for particular polyatomic gases.

1. System of momentum equations. The system of momentum equations for a polyatomic gas can be obtained from the Van Chang–Uhlenbeck kinetic equation by the known procedure [9]. In the case of gas expansion from a spherical source, it includes the equations of conservation of mass, momentum, and total energy, and also equations for $T_{\|}$, $T_{\perp}$ and the average rotational energy $E_r$:

$$\frac{d}{dr}(nur^2) = 0, \qquad nu\frac{du}{dr} + \frac{d}{dr}\Big(\frac{nkT_{\|}}{m}\Big) + \frac{2}{r}\frac{nk}{m}(T_{\|} - T_{\perp}) = 0,$$
$$nu\frac{d}{dr}\Big(\frac{mu^2}{2} + \frac{3kT_{\|}}{2} + kT_{\perp} + E_r\Big) = 0, \tag{1.1}$$
$$nu^2\frac{du}{dr} + \frac{3}{2}nu\frac{k}{m}\frac{dT_{\|}}{dr} - \frac{2}{r}nu\frac{kT_{\perp}}{m} = R_{\|}, \qquad nu\frac{dE_r}{dr} = R_r.$$

The equation for $T_{\perp}$ does not enter the complete system of equations, because it is not independent (summing it up with the equations for $T_{\|}$ and $E_r$ leads to the equation of conservation of total energy).

System (1.1) can be obtained using an ellipsoidal representation for the distribution function of translational degrees of freedom (see, for example, [1]) and the Boltzmann distribution of rotational degrees of freedom:

$$f_i = \frac{n}{Q}\Big(\frac{m}{2\pi kT_{\|}}\Big)^{1/2}\Big(\frac{m}{2\pi kT_{\perp}}\Big)\exp\Big[-\frac{m}{2kT_{\|}}(\xi - u)^2 - \frac{m}{2kT_{\perp}}\rho^2 - \varepsilon_i\Big]. \tag{1.2}$$

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Here $Q = \sum_i \exp(-\varepsilon_i)$; $\varepsilon_i = E_i/kT_r$ ($E_i$ is the energy of the $i$th rotational level). In this case,

$$E_r = kT_r\langle\varepsilon(T_r)\rangle, \qquad dE_r = c_r(T_r)\,dT_r, \tag{1.3}$$

where $\langle\varepsilon(T_r)\rangle = Q^{-1}\sum_i \varepsilon_i \exp(-\varepsilon_i)$; $c_r(T_r)$ is the heat capacity of the rotational degrees of freedom.

The momenta of the integral of collisions $R_\parallel$ and $R_r$ using (1.2) were calculated by Randenija and Smith [8]. They are complex functions of $T_\parallel$, $T_\perp$ and $T_r$. Because of this, system (1.1) can be solved only numerically under certain restrictions. To perform a step-by-step asymptotic analysis of system (1.1), which makes it possible to obtain analytical results, we use the approximation of a weak nonequilibrium jet:

$$\frac{T_\parallel - T}{T} \ll 1, \qquad \frac{T_\perp - T}{T} \ll 1, \qquad \frac{T_r - T}{T} \ll 1. \tag{1.4}$$

Here the temperature $T$ corresponds to the equilibrium energy of the jet, and is given by the relation

$$\frac{1}{2}kT_\parallel + kT_\perp + E_r(T_r) = \frac{5}{2}kT + E_r(T). \tag{1.5}$$

Note that, with satisfaction of conditions (1.4), we have $E_r(T_r) - E_r(T) \simeq c_r(T_r - T)$. Calculation of $R_\parallel$ and $R_r$ using (1.2) and taking into account (1.4) and (1.5) gives

$$R_\parallel = -\frac{1}{3}n\frac{k(T_\parallel - T_\perp)}{\tau} + \frac{1}{3}n\frac{c_r}{\tau_E}\Big[(T_r - T_\perp) - \frac{2}{5}\Big(\frac{3}{2} - \varepsilon_0\Big)(T_\parallel - T_\perp)\Big],$$

$$R_r = -n\frac{c_r}{\tau_E}\Big[(T_r - T_\perp) - \frac{1}{3}(T_\parallel - T_\perp)\Big].$$

In this case, $\tau^{-1}$ and $\tau_E^{-1}$ are the characteristic frequencies of elastic and inelastic collisions of molecules, given by the relations [9]

$$\tau^{-1} = \frac{8}{5}n\langle\gamma^4(1-\cos^2\chi)\rangle_c, \qquad \tau_E^{-1} = \frac{2k}{c_r}n\langle(\Delta\varepsilon)^2\rangle_c$$

$$\Big(\langle F\rangle_c = 2\pi\Big(\frac{kT}{\pi m}\Big)^{1/2}Q^{-2}\sum_{ijkl}\int F\gamma^3\exp(-\gamma^2 - \varepsilon_i - \varepsilon_j)\sigma_{ij}^{kl}(g,\chi)\sin\chi\,d\chi\,d\gamma\Big).$$

Furthermore, $\varepsilon_0 = \langle(\Delta\varepsilon)^2\sin^2\chi\rangle_c/\langle(\Delta\varepsilon)^2\rangle_c$. Here $\gamma = (m/4kT)^{1/2}g$; $g$ is the relative particle velocity in collisions, and $\chi$ is the scattering angle.

In what follows, instead of the equations for $T_\parallel$, $T_\perp$, and $E_r$, it is convenient to use the equations for $T_\parallel$, $T_\perp$, and $T$. Taking into account (1.4), from condition (1.5) we obtain $c_r(T_r - T) \simeq -(1/2)k(T_\parallel - T) - k(T_\perp - T)$. Using this relation to eliminate $T_r$ in the system of momentum equations and in the formulas for $R_\parallel$ and $R_r$ and taking into account that $R_\parallel + R_\perp + R_r = 0$, we obtain the resulting system of momentum equations

$$\frac{d}{dr}(nur^2) = 0, \quad nu\frac{du}{dr} + \frac{d}{dr}\Big(\frac{nkT_\parallel}{m}\Big) + \frac{2}{r}\frac{nk}{m}(T_\parallel - T_\perp) = 0, \quad uc_V\frac{dT}{dr} + kT_\parallel\frac{du}{dr} + \frac{2ukT_\perp}{r} = 0,$$

$$\frac{1}{2}uk\frac{dT_\parallel}{dr} + kT_\parallel\frac{du}{dr} = -\frac{k}{\tau}[C(Z)(T_\parallel - T) - A(Z)(T_\perp - T)], \tag{1.6}$$

$$uk\frac{dT_\perp}{dr} + \frac{2ukT_\perp}{r} = \frac{k}{\tau}[A(Z)(T_\parallel - T) - B(Z)(T_\perp - T)].$$

Here the rotational number of collisions $Z = (4/\pi)\tau_E/\tau$,

$$A(Z) = \frac{1}{3}\Big\{1 - \frac{8}{5}\Big[\frac{5}{2} + \frac{c_r}{k}(1+\varepsilon_0)\Big](\pi Z)^{-1}\Big\},$$

$$B(Z) = \frac{1}{3}\Big\{1 + \frac{8}{5}\Big[5\Big(1 + \frac{c_r}{k}\Big) - \frac{c_r}{k}(1+\varepsilon_0)\Big](\pi Z)^{-1}\Big\}, \tag{1.7}$$

$$C(Z) = \frac{1}{3}\Big\{1 + \frac{8}{5}\Big[\frac{5}{4} + \frac{c_r}{k}\Big(\frac{3}{2} - \varepsilon_0\Big)\Big](\pi Z)^{-1}\Big\},$$

and, in addition, $c_V(T) = (3/2)k + c_r(T)$.

For the subsequent asymptotic analysis of the system of momentum equations (1.6), it is necessary to specify the dependences of the functions $\tau_t$, $Z$, $c_r$ and $\varepsilon_0$ on $T$. The value of $\tau_E$ is essentially determined by the anisotropic (inelastic) part of the interaction potential, while $\tau_t = \eta/p$ ($\eta$ is the shear viscosity of the gas [9]) is found from the assumption of purely elastic scattering [10]. In specific calculations of jet relaxation, the values of $\tau_t$ and $\varepsilon_0$ are usually determined using the part of the Lennard-Jones potential that corresponds to the attraction branch $[V(r) = -C_6/r^6]$. This gives an adequate description of the elastic scattering of molecules at low energies, which correspond to the conditions of jet expansion [11]. Then, for the frequency of elastic scattering and $\varepsilon_0$ we have

$$\tau_t^{-1} = 6.82n\left(\frac{kT}{m}\right)^{1/2}\left(\frac{C_6}{kT}\right)^{1/3} \tag{1.8}$$

[8] and $\varepsilon_0 = 0.581$ [6]. In addition, we assume that $c_r(T) = (1/2)jk$, where $j$ is the number of rotational degrees of freedom. This is valid for not too low temperatures. We also assume that $Z = \text{const}$. The latter assumption was discussed in [8]. Note that the last two assumptions are not obligatory for asymptotic analysis. It is also possible to use more complex dependences of $c_r$ and $Z$ on $T$, for example, exponential dependences.

Under these assumptions, the system of momentum equations takes the form

$$\begin{gathered}
nur^2 = \left(\frac{5+j}{3+j}\right)^{1/2}\mathrm{M}_s, \qquad nu\frac{du}{dr} + \frac{d}{dr}(nT_\parallel) + \frac{2n}{r}(T_\parallel - T_\perp) = 0,\\
\frac{3+j}{2}u\frac{dT}{dr} + T_\parallel\frac{du}{dr} + \frac{2uT_\perp}{r} = 0,\\
\frac{1}{2}u\frac{dT_\parallel}{dr} + T_\parallel\frac{du}{dr} = -A_s nT^{1/6}[C(Z)(T_\parallel - T) - A(Z)(T_\perp - T)],\\
u\frac{dT_\perp}{dr} + \frac{2uT_\perp}{r} = A_s nT^{1/6}[A(Z)(T_\parallel - T) - B(Z)(T_\perp - T)],\\
A(Z) = \frac{1}{3}[1 - 0.402(j + 3.163)Z^{-1}], \qquad B(Z) = \frac{1}{3}[1 + 0.870(j + 2.925)Z^{-1}],\\
C(Z) = \frac{1}{3}[1 + 0.234(j + 2.720)Z^{-1}],
\end{gathered} \tag{1.9}$$

where $\mathrm{M}_s$ is the Mach number for the source $(r = r_s)$ $A_s = 4/(5\sqrt{\pi})\mathrm{Kn}_s^{-1}\Gamma(11/3)$; $\mathrm{Kn}_s^{-1} = 3.770\, n_s r_s (C_6/kT_s)^{1/3}$.

The dimensionless system of momentum equations (1.9) is obtained from (1.6) by introduction of the following scales for variables: $n_s$ for density, $T_s$ for temperature, $(kT_s/m)^{1/2}$ for velocity, and $r_s$ for distance (the subscript $s$ corresponds to the source conditions). The parameter $\mathrm{Kn}_s$ (the Knudsen number for the source) is usually small in jet experiments.

**2. Solution for the Internal Region.** Using the smallness of the Knudsen number for the source ($\mathrm{Kn}_s \ll 1$), we shall seek a solution for system (1.9) for the jet parameters near the nozzle in the form

$$\Gamma(r, A_s) = \Gamma_0(r) + A_s^{-1}\Gamma_1(r) + \dots\,. \tag{2.1}$$

As a zero approximation, we have

$$\begin{gathered}
n_0u_0r^2 = \left(\frac{5+j}{3+j}\right)^{1/2}\mathrm{M}_s,\\
u_0^2 + (3+j)\left(\frac{5+j}{3+j}\right)^{(4+j)/(3+j)}\mathrm{M}_s^{2/(3+j)}u_0^{-2/(3+j)}r^{-4/(3+j)} = \left(\frac{5+j}{3+j}\right)(\mathrm{M}_s^2 + 3 + j),\\
T_0 = n_0^{2/(3+j)}, \qquad T_{r0} = T_{\parallel 0} = T_{\perp 0} = T_0,
\end{gathered} \tag{2.2}$$

i.e., the usual continual solution for the spherically symmetrical expansion of monatomic gas.

As $r \to \infty$, from (2.2) it follows that

$$n_0 \simeq \mathrm{M}_s(\mathrm{M}_s^2+3+j)^{-1/2}r^{-2}+\ldots,\quad u_0 \simeq \left(\frac{5+j}{3+j}\right)^{1/2}(\mathrm{M}_s^2+3+j)^{1/2}+\ldots,$$
$$T_0 \simeq \mathrm{M}_s^{2/(3+j)}(\mathrm{M}_s^2+3+j)^{-1/(3+j)}r^{-4/(3+j)}+\ldots. \tag{2.3}$$

Next, using (2.3) in one of the last two equations of system (1.9) and comparing terms of the zero and first approximations, we can show that the uniform applicability of expansion (2.1) is violated at distances

$$r = O(A_s^{3(3+j)/(3j+11)}). \tag{2.4}$$

This determines the external boundary of the internal flow region. In going over to the solution of the system of momentum equations in the external region, we should rescale the equations.

**3. Solution for the External Region.** In rescaling the equations, we use the fact that the dimension of the internal region is determined by condition (2.4) and the behavior of the parameters at the boundary (2.3). We introduce new variables:

$$r = s_1 A_s^{3(3+j)/(3j+11)},\quad n = NA_s^{-6(3+j)/(3j+11)},\quad T=\tau A_s^{-12/(3j+11)},\quad u=U. \tag{3.1}$$

Then, the system of momentum equations in the external region takes the form

$$\frac{d}{ds_1}(NUs_1^2)=0; \tag{3.2}$$

$$NU\frac{dU}{ds_1}+A_s^{-12/(3j+11)}\left[\frac{d}{ds_1}(N\tau_\parallel)+\frac{2N}{s_1}(\tau_\parallel-\tau_\perp)\right]=0; \tag{3.3}$$

$$\frac{3+j}{2}U\frac{d\tau}{ds_1}+\tau_\parallel\frac{dU}{ds_1}+\frac{2U\tau_\perp}{s_1}=0; \tag{3.4}$$

$$\frac{1}{2}U\frac{d\tau_\parallel}{ds_1}+\tau_\parallel\frac{dU}{ds_1}=-[C(Z)(\tau_\parallel-\tau)-A(Z)(\tau_\perp-\tau)]N\tau^{1/6}; \tag{3.5}$$

$$U\frac{d\tau_\perp}{ds_1}+\frac{2U\tau_\perp}{s_1}=[A(Z)(\tau_\parallel-\tau)-B(Z)(\tau_\perp-\tau)]N\tau^{1/6}. \tag{3.6}$$

In addition, $\tau_r = ((j+3)/j)\tau-(1/j)\tau_\parallel-(2/j)\tau_\perp$

The external limit of the internal asymptotic expansion in the new variables is

$$U=\left(\frac{5+j}{3+j}\right)^{1/2}(\mathrm{M}_s^2+3+j)^{1/2}+\ldots,\qquad N=\mathrm{M}_s(\mathrm{M}_s^2+3+j)^{-1/2}s_1^{-2}+\ldots,$$
$$\tau_0=\mathrm{M}_s^{2/(3+j)}(\mathrm{M}_s+3+j)^{-1/(3+j)}s_1^{-4/(3+j)}+\ldots. \tag{3.7}$$

The structure of system (3.2)–(3.6) shows that its solution should be sought in the form $\Gamma(s_1,A_s)=\Gamma_0(s_1)+A_s^{-12/(3j+11)}\Gamma_1(s_1)+\ldots$. As a zero approximation, we obtain

$$U_0=\left(\frac{5+j}{3+j}\right)^{1/2}(\mathrm{M}_s^2+3+j)^{1/2}; \tag{3.8}$$

$$N_0=\left(\frac{5+j}{3+j}\right)\mathrm{M}_s^{-1}(\mathrm{M}_s^2+3+j)^{3/2}s^{-2}; \tag{3.9}$$

$$\frac{d\tau_0}{ds}=-\frac{4}{3+j}\frac{\tau_{\perp0}}{s}; \tag{3.10}$$

$$\frac{1}{2}s^2\frac{d\tau_{\parallel0}}{ds}=-[C(Z)(\tau_{\parallel0}-\tau_0)-A(Z)(\tau_{\perp0}-\tau_0)]\tau_0^{1/6}; \tag{3.11}$$

$$s^2\frac{d\tau_{\perp0}}{ds}+2s\tau_{\perp0}=[A(Z)(\tau_{\parallel0}-\tau_0)-B(Z)(\tau_{\perp0}-\tau_0)]\tau_0^{1/6},$$
$$\tau_{r0}=\frac{j+3}{j}\tau_0-\frac{1}{j}\tau_{\parallel0}-\frac{2}{j}\tau_{\perp0}. \tag{3.12}$$

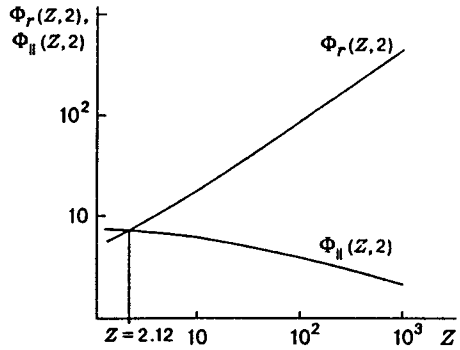

Fig. 1. Dependences $\Phi_{\|}(Z,2)$ and $\Phi_r(Z,2)$ on $Z$.

Here we introduce a new variable: $s = ((5+j)/(3+j))^{1/2}\mathrm{M}_s^{-1}(\mathrm{M}_s^2+3+j)s_1$. As $s \to 0$, the solution of system (3.8)–(3.12) has the form

$$\tau_{r0} = \tau_{\|0} = \tau_{\perp 0} = \tau_0 = Ks^{-4/(3+j)}. \tag{3.13}$$

"Sewing" with (3.7) gives the value of the constant

$$K = \left(\frac{5+j}{3+j}\right)^{2/(3+j)} \mathrm{M}_s^{-2/(3+j)}(\mathrm{M}_s^2+3+j)^{3/(3+j)}. \tag{3.14}$$

Furthermore, it is easy to show that as $s \to \infty$, system (3.8)–(3.12) admits the following solution:

$$\tau_{\|0}(s) = \tau_{\|0}(\infty) + 2\tau_0^{1/6}(\infty)\{\tau_{\|0}(\infty)C(Z) + [A(Z)-C(Z)]\tau_0(\infty)\}\frac{1}{s},$$

$$\tau_{\perp}(s) = \tau_0^{1/6}(\infty)\{\tau_{\|0}(\infty)A(Z) + [B(Z)-A(Z)]\tau_0(\infty)\}\frac{1}{s},$$

$$\tau_0(s) = \tau_0(\infty) + \frac{4}{3+j}\tau_0^{1/6}(\infty)\{\tau_{\|0}(\infty)A(Z) + [B(Z)-A(Z)]\tau_0(\infty)\}\frac{1}{s}.$$

Hence

$$\tau_{r0}(s) = \tau_{r0}(\infty) + \frac{2}{j}\tau_0^{1/6}(\infty)\{\tau_{\|0}(\infty)[A(Z)-C(Z)] + \tau_0(\infty)[B(Z)+C(Z)-2A(Z)]\}\frac{1}{s},$$

where $\tau_{\|0}(\infty)$, $\tau_{\perp 0}(\infty)$, and $\tau_0(\infty)$ are found by numerical integration of system (3.10)–(3.12) with boundary conditions (3.13) and (3.14), and they depend on $Z$ and $\mathrm{M}_s$.

Reverting to dimensional variables, we can write expressions for the limiting parallel and rotational temperatures of a polyatomic gas relating them with the source conditions:

$$T_{r(\|)\infty} = (9.690\cdot 10^{27})^{-12/(3j+11)}\Phi_{r(\|)}(Z,j)\frac{(T_0^0)^{3(j+9)/(3j+11)}}{(p_0^0 D)^{12/(3j+11)}C_6^{4/(3j+11)}}.$$

Here $\Phi_{r(\|)}(Z,j) = \tau_{r(\|)0}(\infty)[r_s(1+(1/(3+j))\mathrm{M}_s^2)^{-(j+1)/4}]^{-12/(3j+11)}$ is a function that depends only on $Z$ and $j$, because the real temperature should not depend on the location of "sewing" $r = r_s$; $p_0^0$ and $T_0^0$ in the nozzle are given in atm and K, respectively, and $D$ is given in cm.

**4. Results and Discussion.** Particular calculations were performed for diatomic gases ($j = 2$). In this case, expressions for the limiting parallel and rotational temperatures have the form

$$T_{\|\infty} = 1.758\cdot 10^{-20}\Phi_{\|}(Z,2)\frac{(T_0^0)^{33/17}}{(p_0^0 D)^{12/17}C_6^{4/17}}; \tag{4.1}$$

$$T_{r\infty} = 1.758\cdot 10^{-20}\Phi_r(Z,2)\frac{(T_0^0)^{33/17}}{(p_0^0 D)^{12/17}C_6^{4/17}}. \tag{4.2}$$

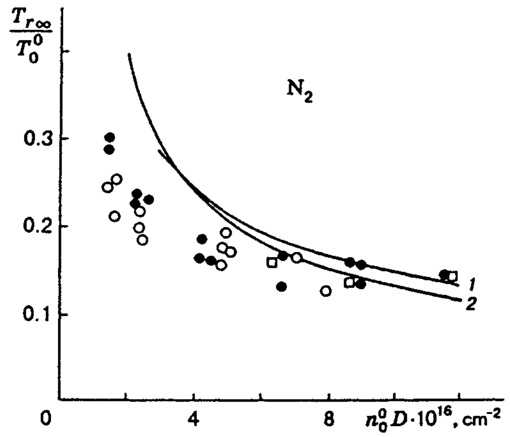


Fig. 2

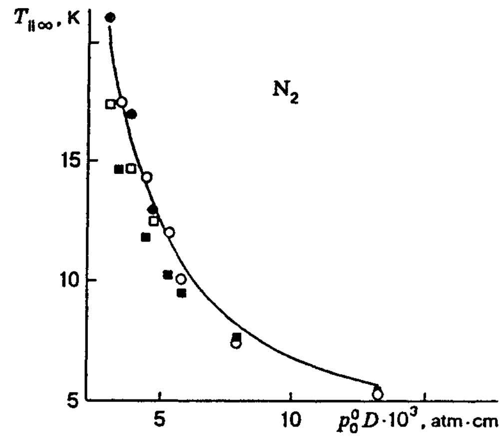


Fig. 3

Fig. 2. Dependence of $T_{r\infty}/T_0^0$ on $n_0^0 D$: points • refers to the experiment of [12] $T = 300$ K, points o refers to the experiment of [12] $T = 600$ K, curve 1 refers to the theory of [12], curve 2 refers to the calculation using formula (4.2) of the present paper for $Z = 9$; points □ refer to the calculation of [7] for the data of [12].

Fig. 3. Dependence of $T_{\|\infty}$ on $p_0^0 D$: points • refer to the experiment of [12], points o refer to the experiment of [16], the curve refers to the calculation using formula (4.1) of the present paper for $Z = 9$; points □ refer to the calculation of [7] for the data of [12], and points ■ refer to the calculation of [7] for the data of [16].

Figure 1 shows the dependences of $\Phi_r(Z,2)$ and $\Phi_\|(Z,2)$ on $Z$. It is evident that $\Phi_\|(Z,2)$ depends relatively weakly on $Z$, while $\Phi_r(Z,2)$ increases fairly rapidly with increase in $Z$. Note that, at large $Z$, the given dependences are well approximated by the expressions $\Phi_r(Z,2) = 3.427Z^{12/17}$ and $\Phi_\|(Z,2) = 16.490Z^{-5/17}$. The character of the asymptotic expressions agrees with the estimates of [5]. In addition, note that, for $Z = 2.12$, we have $\Phi_r(Z,2) = \Phi_\|(Z,2)$, i.e., the limiting parallel and rotational temperatures are equal under any source conditions. This behavior of the temperatures was noted in [5, 6].

Figure 2 gives a comparison of the calculated and experimental dependences of the limiting rotational temperature on $n_0^0 D$ for $N_2$ using the experimental results of Poulsen and Miller [12]. Curve 1 was calculated in [12] using the linear equation of relaxation of rotational energy and the assumption of isentropic behavior of the jet parameters. Good agreement with the experimental dependence for $Z_r = 3$ was obtained in [12]. Note the significant difference between the definition of the parameter $Z_r$ in [12] and the definition of the parameter $Z$ in the present paper. In [12], as in many other papers (see, for example, [7, 13–15]), $Z_r$ is given by the relation $Z_r = \tau_r/\tau_t = \tau_r \nu$, where $\nu = \sqrt{2}n\langle v\rangle\pi\sigma^2$ is the average frequency of elastic collisions for rigid spheres [9]. This relation does not take into account the real behavior of the frequency of elastic collisions (which is given by relation (1.8) for the potential $V(r) = -C_6/r^6$) at low temperatures and leads to underestimated values of the experimental value of $Z$. The calculation by formula (4.2) gives curve 2, which describes satisfactorily the experimental data for $Z = 9$.

Figure 3 compares the experimental and calculated dependences of $T_{\|\infty}$ on $p_0^0 D$. The solid curve is the calculation by formula (4.1) for $Z = 9$ and $C_6 = 1.256\cdot 10^{-58}$ erg·cm$^6$ [17]. For comparison, Figs. 2 and 3 give the results of numerical solution of the system of momentum equations with the linear equation of relaxation of rotational energy (weak nonequilibrium for rotational degrees of freedom) [7] for the conditions of [12, 16]. As can be seen from the above comparison of the theoretical and experimental curves, the dependences of $T_r$ on $p_0^0 D$ can serve to determine the rotational number of collisions at low temperatures. Note, however, that, in this case, one should take into account the difference between the theory and experiment at small values of $p_0^0 D$. This difference is possibly caused by the nonequilibrium for both translational and rotational degrees of freedom, which increases with decrease in $p_0^0 D$. The latter circumstance was noted, for example, in [18, 19].

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